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Zusatzübungen 05 zum mathematischen Vorkurs der MVHS

1. Bestimmen Sie jeweils die Lösungsmenge der folgenden Gleichungen

   (a)
   $$\frac{x^2}{x^3} = x^{-1} \quad \Rightarrow \quad \mathbb{D} = \mathbb{R}^* = \mathbb{L}$$

   (b)
   $$x^2 + 36x^{-2} = 13 \quad \Rightarrow \quad \mathbb{D} = \mathbb{R}^* \qquad \mathbb{L} = \{-3; -2; 2; 3\}$$

   (c)
   $$729x^6 = 1 \quad \Rightarrow \quad \mathbb{D} = \mathbb{R} \qquad \mathbb{L} = \left\{-\frac{1}{3}; \frac{1}{3}\right\}$$

   (d)
   $$\frac{13x+3}{7x+1} = \frac{15}{8} \quad \Rightarrow \quad \mathbb{D} = \mathbb{R} \setminus \left\{-\frac{1}{7}\right\} \qquad \mathbb{L} = \{9\}$$

   (e)
   $$\frac{7x+5}{15x-11} = \frac{7x-5}{15x-27} \quad \Rightarrow \quad \mathbb{D} = \mathbb{R} \setminus \left\{\frac{11}{15}; \frac{9}{5}\right\} \qquad \mathbb{L} = \{5\}$$

   (f)
   $$\frac{\frac{x}{x+1}}{\frac{1}{x} - \frac{1}{x+1}} = \frac{\frac{x+1}{x}}{\frac{1}{x-1} - \frac{1}{x}} \quad \Rightarrow \quad \mathbb{D} = \mathbb{R} \setminus \{\pm 1; 0\} \qquad \mathbb{L} = \{\ \}$$

   (g)
   $$3 \cdot \sqrt{3x+1} - 2x = 6 - \sqrt{3x+1} \quad \Rightarrow \quad \mathbb{D} = \left[-\frac{1}{3}; \infty\right[\qquad \mathbb{L} = \{1; 5\}$$

   (h)
   $$\sqrt{5x-1} - 2\sqrt{3-x} = \sqrt{x-1} \quad \Rightarrow \quad \mathbb{D} = [1; 3] \qquad \mathbb{L} = \{2\}$$

   (i)
   $$x + 5\sqrt{x} = 14 \quad \Rightarrow \quad \mathbb{D} = \mathbb{R}_0^+ \qquad \mathbb{L} = \{4\}$$

   (j)
   $$x^4 + 12 = 7x^2 \quad \Rightarrow \quad \mathbb{D} = \mathbb{R} \qquad \mathbb{L} = \left\{\pm 2; \pm\sqrt{3}\right\}$$

   (k)
   $$4^{2x-3} \cdot 32^{1-x} = \frac{1}{8} \quad \Rightarrow \quad \mathbb{D} = \mathbb{R} \qquad \mathbb{L} = \{2\}$$

   (l)
   $$\log_4(x-1) - \log_4(3x-5) = 1 - \log_4(x) \quad \Rightarrow \quad \mathbb{D} = \left]\frac{5}{3}; \infty\right[\qquad \mathbb{L} = \left\{\frac{13 \pm \sqrt{89}}{2}\right\}$$

2. Bestimmen Sie jeweils die Definitions- und Lösungsmenge der folgenden Gleichungen

   (a)
   $$3^x + 9^x = \sqrt{3}\left(\sqrt{3}+1\right)\ ,\ \mathbb{D} = \mathbb{R}\,,\ \mathbb{L} = \left\{\frac{1}{2}\right\}$$

   (b)
   $$\left(\frac{1}{2}\right)^{x-1} \cdot \left(8^{x+1} - 4^5\right) = 16\left(4^x - 8\right)\ ,\ \mathbb{D} = \mathbb{R}\,,\ \mathbb{L} = \{4\}$$

   (c)
   $$\log_9\left(1 + \log_2(x)\right) = \log_3(2)\ ,\ \mathbb{D} =]0,5\,;\,\infty[\,,\ \mathbb{L} = \{8\}$$