Ableitungen von Funktionen

Bestimmen Sie die ersten und zweiten Ableitungen zu folgenden Funktionen

Lösungen:

1.
$$\begin{aligned} f(x) &= \sin(x) \\ f'(x) &= \cos(x) \\ f''(x) &= -\sin(x) \end{aligned}$$

2.
$$\begin{aligned} f_1(x) &= \sin(2x) \\ f_1'(x) &= 2\cos(2x) \\ f_1''(x) &= -4\sin(2x) \end{aligned}$$

3.
$$\begin{aligned} f_2(x) &= \sin(2x-3) \\ f_2'(x) &= 2\cos(2x-3) \\ f_2''(x) &= -4\sin(2x-3) \end{aligned}$$

4.
$$\begin{aligned} f_3(x) &= \sin(x^2) \\ f_3'(x) &= 2x\cos(x^2) \\ f_3''(x) &= 2\cos(x^2) - (2x)^2\sin(x^2) \end{aligned}$$

5.
$$\begin{aligned} f_4(x) &= \sin(4x^2) \\ f_4'(x) &= 8x\cos(4x^2) \\ f_4''(x) &= 8\cos(4x^2) - (8x)^2\sin(4x^2) \end{aligned}$$

6.
$$\begin{aligned} f_5(x) &= \sin(4x^2+2x-3) \\ f_5'(x) &= (8x+2)\cos(4x^2+2x-3) \\ f_5''(x) &= 8\cos(4x^2+2x-3) - (8x+2)^2\sin(4x^2+2x-3) \end{aligned}$$

7.

$$
\begin{aligned}
f_6(x) &= \sin(\sqrt{4x^2+2x-3}-\sqrt{x}) \\
f_6'(x) &= \left(\frac{8x+2}{2\sqrt{(4x^2+2x-3)}}-\frac{1}{2\sqrt{x}}\right)\cos(\sqrt{4x^2+2x-3}-\sqrt{x}\,) \\
f_6''(x) &= -\sin(\sqrt{4x^2+2x-3}-\sqrt{x}\,)\cdot\left(\frac{8x+2}{2\sqrt{(4x^2+2x-3)}}-\frac{1}{2\sqrt{x}}\right)^2 \\
&\quad +\cos(\sqrt{4x^2+2x-3}-\sqrt{x}\,)\frac{1}{2} \\
&\quad \cdot\left(-\frac{1}{2}(4x^2+2x-3)^{-\frac{3}{2}}(8x+2)^2+(4x^2+2x-3)^{-\frac{1}{2}}8+\frac{1}{2}x^{-\frac{3}{2}}\right)
\end{aligned}
$$

1.

$$\begin{aligned} h_1(x) &= \frac{1}{x} \\ h_1'(x) &= -\frac{1}{x^2} \\ h_1''(x) &= \frac{2}{x^3} \end{aligned}$$

2.

$$\begin{aligned} h_2(x) &= \frac{1}{2x} \\ h_2'(x) &= -\frac{1}{2x^2} \\ h_2''(x) &= \frac{1}{x^3} \end{aligned}$$

3.

$$\begin{aligned} h_3(x) &= \frac{1}{x^2} \\ h_3'(x) &= -\frac{2}{x^3} \\ h_3''(x) &= \frac{6}{x^4} \end{aligned}$$

4.

$$\begin{aligned} h_4(x) &= \frac{x}{x^2} \\ h_4'(x) &= -\frac{1}{x^2} \\ h_4''(x) &= \frac{2}{x^3} \end{aligned}$$

5.

$$\begin{aligned} h_5(x) &= \frac{x+1}{x^2} \\ h_5'(x) &= -\frac{x+2}{x^3} \\ h_5''(x) &= \frac{2(x+3)}{x^4} \end{aligned}$$

6.

$$
\begin{aligned}
h_6(x) &= \frac{x}{x^2+1} \\
h_6'(x) &= \frac{1-x^2}{(x^2+1)^2} \\
h_6''(x) &= \frac{2x(3+x^2)}{(x^2+1)^3}
\end{aligned}
$$

7.

$$
\begin{aligned}
h_7(x) &= \frac{x+1}{x^2+1} \\
h_7'(x) &= \frac{x^2+2x-1}{(x^2+1)^2} \\
h_7''(x) &= \frac{2(x^3+3x-3x-1)}{(x^2+1)^3}
\end{aligned}
$$

8.

$$
\begin{aligned}
h_8(x) &= \frac{x^2-1}{x^2+1} \\
h_8'(x) &= \frac{4x}{(x^2+1)^2} \\
h_8''(x) &= \frac{4(1-3x^2)}{(x^2+1)^3}
\end{aligned}
$$

9.

$$
\begin{aligned}
h_9(x) &= \frac{5x^2-3x-4}{2x^2+7x+1} \\
h_9'(x) &= \frac{25+26x+41x^2}{(2x^2+7x+1)^2} \\
h_9''(x) &= -\frac{4(81+75x+39x^2+41x^3)}{(2x^2+7x+1)^3}
\end{aligned}
$$