

Grenzwerte von Funktionen

Untersuchen Sie das Verhalten der folgenden Funktionen an den Grenzen ihrer Definitionsbereiche.

Lösungen:

1.

$$\begin{aligned} f(x) = x - 5 &\Rightarrow \mathbb{D} = \mathbb{R} \\ &\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = \pm\infty \end{aligned}$$

2.

$$\begin{aligned} f(x) = x^3 - x^4 + x - 5 &\Rightarrow \mathbb{D} = \mathbb{R} \\ &\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = -\infty \end{aligned}$$

3.

$$\begin{aligned} f(x) = x^{-3} - x^4 + x - 5 &\Rightarrow \mathbb{D} = \mathbb{R} \setminus \{0\} \\ &\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = -\infty \\ &\Rightarrow \lim_{x \rightarrow 0^\pm} f(x) = \pm\infty \end{aligned}$$

4.

$$\begin{aligned} f(x) = \frac{x^2 - x - 5}{x + 4} &\Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-4\} \\ &\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = \mp\infty \\ &\Rightarrow \lim_{x \rightarrow (-4)^\pm} f(x) = \pm\infty \end{aligned}$$

5.

$$\begin{aligned} f(x) = \frac{x^2 - x - 6}{x - 3} &\Rightarrow \mathbb{D} = \mathbb{R} \setminus \{3\} \\ &\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = \pm\infty \\ &\Rightarrow \lim_{x \rightarrow 3^\pm} f(x) = 5 \end{aligned}$$

6.

$$\begin{aligned}f(x) = \frac{x^2 + x - 6}{x^2 - 4} &\Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-2; 2\} \\&\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = 1 \\&\Rightarrow \lim_{x \rightarrow 2^\pm} f(x) = \frac{5}{4} \\&\Rightarrow \lim_{x \rightarrow (-2)^\pm} f(x) = \pm\infty\end{aligned}$$

7.

$$\begin{aligned}f(x) = \frac{3}{x^2 - 1} &\Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-1; 1\} \\&\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = 0 \\&\Rightarrow \lim_{x \rightarrow 1^\pm} f(x) = \pm\infty \\&\Rightarrow \lim_{x \rightarrow (-1)^\pm} f(x) = \pm\infty\end{aligned}$$

8.

$$\begin{aligned}f(x) = \frac{6x - 7}{x^2 + 1} &\Rightarrow \mathbb{D} = \mathbb{R} \\&\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = 0\end{aligned}$$

9.

$$\begin{aligned}f(x) = \ln\left(1 + \frac{1}{x}\right) &\Rightarrow \mathbb{D} =]-\infty; -1[\cup]0; \infty[\\&\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = 0 \\&\Rightarrow \lim_{x \rightarrow -1^-} f(x) = -\infty \\&\Rightarrow \lim_{x \rightarrow 0^+} f(x) = \infty\end{aligned}$$