# Aufgaben Goniometrie

(Aus Barth; „Anschauliche Geometrie“; Klasse 10; Oldenbourg Verlag)

**Vereinfachen Sie:**

**1.** **a)** $\frac{\sin 2\alpha}{\sin \alpha}$ **b)** $\frac{\sin 2\alpha}{\cos \alpha}$

**2.** **a)** $2(\sin \alpha)^2 + (\cos \alpha)^4 - (\sin \alpha)^4$

**b)** $(\sin \alpha + \cos \alpha + 1)(\sin \alpha + \cos \alpha - 1)$

**3.** **a)** $2 \sin (45° + \alpha) \sin (45° - \alpha)$

**b)** $\sin (60° + \alpha) - \sin (60° - \alpha)$

**c)** $\cos (60° + \alpha) + \cos (60° - \alpha)$

**4.** **a)** $\frac{\sin \alpha + \sin 2\alpha}{1 + \cos \alpha + \cos 2\alpha}$ **b)** $\frac{\cos \alpha}{\cos - \sin \alpha} - \frac{\cos \alpha}{\cos \alpha + \sin \alpha}$

**5.** **a)** $\sin (\alpha + \beta) \cos \alpha - \cos (\alpha + \beta) \sin \alpha$

**b)** $\cos (\alpha + \beta) \cos \alpha + \sin (\alpha + \beta) \sin \alpha$

**c)** $\sin (\alpha + \beta) \cos (\alpha - \beta) + \cos (\alpha + \beta) \sin (\alpha - \beta)$

**d)** $\sin (\alpha - \beta) \cos (\beta - \gamma) + \cos (\alpha - \beta) \sin (\beta - \gamma)$

**6.** **a)** $\frac{\sin (\alpha + \beta) - \sin \alpha \cos \beta}{\sin (\alpha + \beta) - \cos \alpha \sin \beta}$ **b)** $\frac{\cos (\alpha + \beta) - \cos \alpha \cos \beta}{\cos (\alpha - \beta) - \sin \alpha \sin \beta}$

**7.** **a)** $\frac{\sin (\alpha + \beta) + \sin (\alpha - \beta)}{\sin (\alpha + \beta) - \sin (\alpha - \beta)}$ **b)** $\frac{\sin (\alpha + \beta) + \sin (\alpha - \beta)}{\cos (\alpha + \beta) + \cos (\alpha - \beta)}$

**c)** $\frac{\cos (\alpha + \beta) - \cos (\alpha - \beta)}{\sin (\alpha + \beta) - \sin (\alpha - \beta)}$

**8.** **a)** $\frac{1 + \cos}{1 - \cos \alpha}$ **b)** $\frac{1 + \cos \alpha}{(\sin \alpha)^2}$ **c)** $\frac{1}{\sin \alpha} + \frac{1}{\tan \alpha}$ **d)** $\frac{1}{\cos \alpha} - \tan \alpha$

(*Tipp:* halbe Winkel!)

**9.** **a)** $\frac{2 \sin \alpha - \sin 2\alpha}{2 \sin \alpha + \sin 2\alpha}$ **b)** $\frac{2 \cos \alpha + \sin 2\alpha}{2 \cos \alpha - \sin 2\alpha}$

**c)** $\frac{\sin \alpha + \sin \alpha \cos \beta}{\sin \beta + \cos \alpha \sin \beta}$ (*Tip:* halbe Winkel!)

**10.** **a)** $\frac{1 - \cos \alpha + \sin \alpha}{1 + \cos \alpha + \sin \alpha}$ **b)** $\frac{1 + \cos \alpha - \sin \alpha}{1 - \cos \alpha - \sin \alpha}$

(*Tipp:* halbe Winkel!)

**11.** **a)** $\frac{\cos 2\alpha}{1 - (\tan \alpha)^2}$ **b)** $\frac{\sin \alpha - \sin \beta}{\cos \alpha + \cos \beta}$ **c)** $\frac{(1 + \cos \alpha) \sin \frac{1}{2} \alpha}{\sin \alpha}$

**d)** $\frac{\cos (\alpha + \beta) \cos (\alpha - \beta)}{\sin (\alpha + \beta) \sin (\alpha - \beta) + (\cos \alpha)^2}$

**Beweisen Sie:**

**1. a)** $\sin\alpha + \sin(\alpha + 120°) + \sin(\alpha + 240°) = 0$

**b)** $\tan\alpha + \tan(\alpha + 120°) + \tan(\alpha + 240°) = 3\tan 3\alpha$

**c)** $\cos\alpha\cos(\alpha + 120°)\cos(\alpha + 240°) = \frac{1}{4}\cos 3\alpha$

**d)** $\tan\alpha\tan(\alpha + 120°)\tan(\alpha + 240°) = -\tan 3\alpha$

**2. a)** $\sin 3x = 3\sin x - 4(\sin x)^3$

**b)** $\cos 3x = 4(\cos x)^3 - 3\cos x$ (*Tipp:* $3x = 2x + x$ !)

**3.** $\tan 3x = \frac{3\tan x - (\tan x)^3}{1 - 3(\tan x)^2}$

**4. a)** $\sin 4x = 8\sin x(\cos x)^3 - 4\sin x\cos x$

**b)** $\cos 4x = 8(\cos x)^4 - 8(\cos x)^2 + 1$

**5.** $\tan 4x = \frac{4\tan x - 4(\tan x)^3}{1 - 6(\tan x)^2 + (\tan x)^4}$

**6. a)** Leite her: $\sin\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}}$ und $\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}}$

**b)** Welche Formel ergibt sich für $\tan\frac{\alpha}{2}$?

**7. a)** $\sin(45° + \alpha) = \cos(45° - \alpha) = \frac{\sin\alpha + \cos\alpha}{\sqrt{2}}$

**b)** $\sin(45° - \alpha) = \cos(45° + \alpha) = \frac{\cos\alpha - \sin\alpha}{\sqrt{2}}$

**c)** $\tan(45° + \alpha) = \frac{1 + \tan\alpha}{1 - \tan\alpha}$

**d)** $\tan(45° - \alpha) = \frac{1 - \tan\alpha}{1 + \tan\alpha}$

**8. a)** $\tan(45° + \alpha) - \tan(45° - \alpha) = 2\tan 2\alpha$

**b)** $\tan(45° + \alpha) + \tan(45° - \alpha) = \frac{2}{\cos 2\alpha}$

**9. a)** $\frac{1 + \sin\alpha}{\cos\alpha} = \frac{\cos\alpha}{1 - \sin\alpha} = \tan\left(45° + \frac{\alpha}{2}\right)$

**b)** $\frac{1 - \sin\alpha}{\cos\alpha} = \frac{\cos\alpha}{1 + \sin\alpha} = \tan\left(45° - \frac{\alpha}{2}\right)$

**10. a)** $\sin 2\alpha = \frac{2\tan\alpha}{1 + (\tan\alpha)^2}$ **b)** $\cos 2\alpha = \frac{1 - (\tan\alpha)^2}{1 + (\tan\alpha)^2}$

**c)** $\tan\alpha = \frac{\sin 2\alpha}{1 + \cos 2\alpha} = \frac{1 - \cos 2\alpha}{\sin 2\alpha}$

**Bestimmen Sie die Lösungsmenge in $[0; 2\pi[$**

1. **a)** $192(\sin x)^2 + 128 \sin x = 75$ **b)** $100(\cos x)^2 + 75 \sin x = 114$
2. **a)** $\sin x = \sin 2x$ **b)** $\frac{1}{4} \sin 2x - \sin x = 0$
   **c)** $4 \sin x \cos x = -\sqrt{2}$ **d)** $\tan 2x + \tan x = 0$
   **e)** $\cos x - \cos 2x = 1$
3. **a)** $\sin x + \cos 2x = 1$ **b)** $\cos x + \cos 2x = 1$
   **c)** $\sin 2x + 2(\cos x)^2 = 1$
- **4.** **a)** $\sin x + \cos x = 0{,}8$ **b)** $8 \sin x + 9 \cos x = 12$
   **c)** $8 \sin x - 9 \cos x = 12$
5. **a)** $(\sin x)^2 + 2 \sin 2x = 3(\cos x)^2$
   **b)** $(\cos x)^2 + 3 \cos 2x = (\sin x)^2$
   **c)** $24(\cos x)^2 - 12(\sin x)^2 = \sin 2x$
   **d)** $6(\sin x)^2 + 8(\cos x)^2 = 7 \sin 2x$
- **6.** **a)** $\sin x = 3 \cos\left(\frac{\pi}{6} - x\right)$ **b)** $\cos x = 3 \sin\left(\frac{\pi}{4} - x\right)$
7. **a)** $15 \cos x = 16 \tan x$ **b)** $6 \sin 2x - 3 \tan x = 5 \sin x$
- **8.** **a)** $\frac{\tan 2x}{\tan x} - \frac{\tan x}{\tan 2x} = 2$ **b)** $2\frac{\tan x}{\tan 2x} + 12\frac{\tan 2x}{\tan x} + 11 = 0$
9. **a)** $\sin 11x = \sin 5x$ **b)** $\cos 13x = \cos 5x$
   **c)** $\sin 7x = \cos 3x$ **d)** $\tan 15x = \tan 9x$
   **e)** $\sin 5x - \sin 3x = \cos 9x - \cos 7x$
10. **a)** $\sin x + \sin 2x + \sin 3x = 0$
    **b)** $\cos x + \cos 2x + \cos 3x = 0$
11. **a)** $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$
    **b)** $\cos x - \cos 2x - \cos 3x - \cos 4x = 0$